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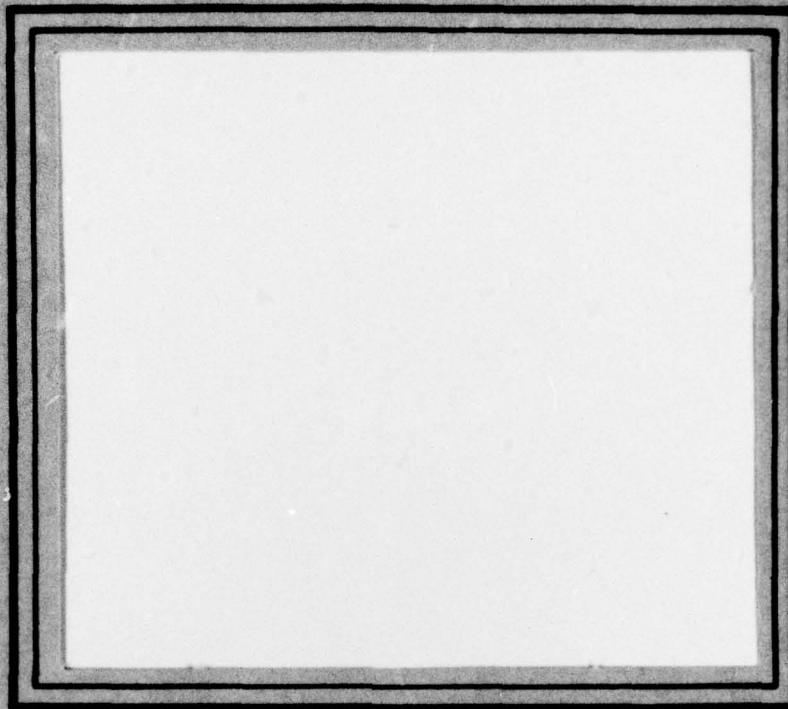


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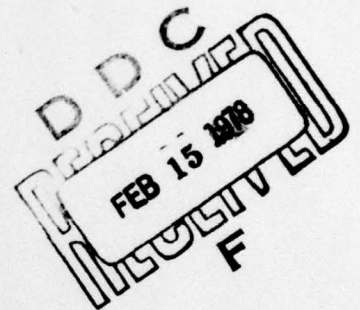


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University of Delaware  
Newark, Delaware

Interim Report on  
Research Grant No. 76-2879, entitled  
Structure of Materials,  
Electromagnetic Scattering  
and Fluid Dynamics\*



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\*October 1, 1976 - October 31, 1977

Principal Investigator: Robert P. Gilbert



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SUMMARY REPORT  
AFOSR Grant No. 76-2879

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R. P. Gilbert and colleagues researched the Riemann-Hilbert boundary value problems for elliptic systems of  $2n$  equations in the plane [1,2]. These problems are of interest in the study of composite laminates. Work on composite elastic materials is in progress by Gilbert and Wendland. Their approach depends heavily on the Begehr-Gilbert representations for solutions. A first report should be available shortly.

Gilbert and A. Jeffrey began joint work on systems of mixed-type (see also, Gilbert-Schneider [3]). These equations appear, for example, in the study of compressible fluid flow in the transonic region.

Gilbert and Hsiao completed a research effort on using finite element methods in connection with integral equations of the first kind to solve boundary value problems associated with elliptic equations [4]. This extends the results developed by Hsiao and Wendland [i,j].

G. C. Hsiao continued his investigations of singular perturbation problems. These researches are listed as manuscripts [9,10] and the book monograph.

Further results concerning numerical and constructive approaches to boundary value problems were made by Gilbert-Vidic [8] who developed an ALGOL program for the accurate computation of Gilbert's G-kernel. Gilbert and Goodrich [5] have investigated Bergman's



kernel function in  $\mathbb{R}^3$ . To do this the regularity of the  $\ell$ -kernel had to be investigated in the closure of the Cartesian product of the respective domain. The  $\ell$ -kernel is not regular on the boundary but is seen to lie in a suitable trace-class.

Other results concerning constructive approaches for metaparabolic equations may be found in [6] and [7].

The Coinvestigator, David Colton, has continued his research on the use of transformation operators to investigate the solutions of parabolic equations with variable coefficients, in particular, these modeling heat conduction in a non-homogeneous medium. In [12] these operators were used to obtain an improved version of Colton's earlier results on reflection principles for parabolic equations. In connection with Lewy's results on reflection laws for elliptic equations, this work now provides a complete picture of reflection properties of solutions to partial differential equations in two independent variables. In [13] Colton's method of transformation operators was used in conjunction with double layer heat potentials to provide a new method for solving initial-boundary value problems for parabolic equations in two independent variables with variable coefficients defined in domains with moving boundaries. Numerical experiments on the use of transformation operators to solve heat conduction problems in a nonhomogeneous medium with moving boundaries were presented in [14] and it was demonstrated that such methods provide an efficient numerical method for solving heat conduction problems of this type.

The Coinvestigator has also continued his investigations on the theory of wave propagation in a spherically stratified medium. In [15] the Coinvestigator's method of transformation operators was used to provide a new method for solving the Fock initial-boundary value problem modeling radiowave propagation around the earth under the assumption of a spherically stratified atmosphere. This new approach leads to the solution of the problem by means of the minimization of a certain quadratic functional, and is based on results on asymptotic analysis due to Erdelyi and the approximation of entire functions due to Levinson. The approximations obtained are valid not only in the shadow and illuminated regions, but in the penumbra region as well. In [16] the problem of the scattering of acoustic waves by an unbounded spherically stratified nonhomogeneous medium was considered, and approximations were obtained which agree with the Born approximation in their common region of validity, but are also applicable for values of the wave number such that the Born approximation is no longer valid. The transformation operator constructed in this paper also strengthens the earlier results of Colton and Wendland on the scattering of acoustic waves by an obstacle in a spherically stratified medium, in the sense that the assumption that the spherically stratified medium had compact support can now be removed.

Surveys of some of the above work can be found in the Coinvestigator's invited paper at the Third Scheveningen Conference on Differential Equations [17] and the invited lecture series at the University of Delaware [18].



MANUSCRIPTS COMPLETED ON AFOSR GRANT NO. 76-2879

During the period October 1, 1976 - October 1, 1977, the following papers were written by R. P. Gilbert:

- [1] Begehr, H. and Gilbert, R.P.: "On Riemann boundary value problems for certain linear elliptic systems in the plane," (submitted for publication).
- [2] Begehr, H. and Gilbert, R.P.: "On a class of boundary value problems for a composite system of first order differential equations," (submitted for publication).
- [3] Gilbert, R.P. and Schneider, M.: "On a class of boundary value problems for a composite system of first order differential equations," (submitted for publication).
- [4] Gilbert, R.P. and Hsiao, G.C.: "A new approach for solving elliptic equations with nonconstant coefficients."
- [5] Gilbert, R.P. and Goodrich, R.: "Regularity of the  $\ell$ -kernel and approximations of Bergman's kernel function," (completed).
- [6] Gilbert, R.P. and Hsiao, G.C.: "Higher order metaparabolic equations."
- [7] Gilbert, R.P. and Roach, G.F.: "Constructive methods for meta and pseudoparabolic systems," Bull. Calcutta Math. Soc. (to appear).
- [8] Gilbert, R.P. and Vidic, C.: "Näherungslösung zu elliptische Differengleichungen durch Integraloperatoren."

During the same period G. C. Hsiao wrote (in addition to Nos. 4 and 6 above) the following papers:

- [9] Hsiao, G.C. and Roach, G.: "On the relationship between boundary value problems," (to appear).
- [10] Hsiao, G.C.: "Singular perturbations of an exterior Dirichlet problem," SIAM J. Math. Anal. (to appear).
- [11] Hsiao, G.C. and Weinacht, R.J.: "On a class of singular perturbation problems for a class of singular partial differential equations," (submitted).



Manuscripts completed  
on Grant No. 76-2879 (cont.)

- [12] Colton, D.: "On reflection principles for parabolic equations in one space variable" (submitted for publication).
- [13] Colton, D.: "The solutions of initial-boundary value problems for parabolic equations by the method of integral operators," J. Diff. Eqns. (to appear).
- [14] Chang, Y.F. and Colton, D.: "The numerical solution of parabolic partial differential equations by the method of integral operators," Int. J. Computer Math., (to appear).
- [15] Colton, D.: "The solution of problems in radiowave propagation by the method of parabolic equations and transformation operators," Appl. Anal. (to appear).
- [16] Colton, D. and Kress, R.: "The construction of solutions to acoustic scattering problems in a spherically stratified medium," Quart. J. Mech. Appl. Math. (to appear).
- [17] Colton, D.: "Transformation operators and wave propagation in a spherically stratified medium," in Third Scheveningen Conf. on Differential Equations, North-Holland Publishing Co., Amsterdam (to appear).
- [18] Colton, D.: Integral Operator Methods in the Theory of Wave Propagation and Heat Conduction, Lecture Notes, Institute for Mathematical Sciences, University of Delaware, 1977.

Manuscripts completed  
on Grant No. 76-2879 (cont.)

The following manuscripts appeared in print during this time:

- (a) Gilbert, R.P.: "Nonlinear boundary value problems for elliptic systems in the plane," Nonlinear Systems and Applications, ed. V. Lakshmikantham, Academic Press, 1977.
- (b) Bhatnagar, S.C. and Gilbert, R.P.: "A method of ascent for a class of sixth order elliptic equations," Bull. Math. de la Soc. Sci. Math. R.S. de Roumanie, Tome 18(66), 1977.
- (c) Brown, P.M. and Gilbert, R.P.: "Constructive methods for higher order, analytic, Sobolev-Galpern equations," Bull. Math. Soc. R.S. de Roumanie, Tome 19(67), 1977.
- (d) Gilbert, R.P. and Weinacht, R.J.: Function Theoretic Methods in Differential Equations (edited), Pitman Publ., London, 1977.
- (e) Gilbert, R.P. and Hile, Gerald: "Hypercomplex Function Theory in the Sense of L. Bers," Math. Nach., Band 72(1976).
- (f) Gilbert, R.P. and Hile, G.N.: "Hilbert function modules with reproducing kernels," Nonlinear Analysis, 1, 2(1977).
- (g) Gilbert, R.P. and Hile, G.N.: "Reproducing kernels for generalized Hilbert function spaces," Appl. Anal. 6,2(1977).
- (h) Begehr, Heinrich and Gilbert, R.P.: "Randwertaufgaben ganzzahliger Charakteristik fur verallgemeinerte hyperanalytische Funktionen," Appl. Anal. 6,2(1977).
- (i) Hsiao, G.C. and Wendland, W.: "A finite element method for integral equations of the first kind," J. Math. Anal. Appl. 68, 3(1977), 449-481.
- (j) Hsiao, G.C. and Wendland, W.: "Galerkin's method for a class of integral equations of the first kind," Appl. Anal. 6 (1977), 155-157.
- (k) Hsiao, G.C.: "On a two dimensional boundary value problem of elasticity," SIAM J. Appl. Math. 31, 4(1976), 677-685.
- (l) Ho, T.C. and Hsiao, G.C.: "Estimate of the effectiveness of a cylindrical catalyst support: a singular perturbation approach," Chem. Eng. Sci., 32(1977), 63-66.

Manuscripts completed  
on AFOSR Grant No. 76-2879

- (m) Colton, David: "An inverse scattering problem for acoustic waves in a spherically symmetric medium," Proc. Edinburgh Math. Soc., 20(1977), 257-263.
- (n) Colton, David: "The scattering of acoustic waves by a spherically stratified inhomogeneous medium," Proc. Royal Soc. Edinburgh, 76A(1977), 345-350.
- (o) Colton, David and Wendland, W.: "Constructive methods for solving the exterior Neumann problem for the reduced wave equation in a spherically symmetric medium," Proc. Royal Soc. Edinburgh, 75A(1976), 98-107.
- (p) Colton, David: "The approximation of solutions to initial boundary value problems for parabolic equations in one space variable," Quart. Applied Math., 33(1976), 377-386.
- (q) Colton, David: "Integral operators and inverse problems in scattering theory," in Proceedings of the Symposium on Function Theoretic Methods for Partial Differential Equations, Springer-Verlag Lecture Note Series, Vol. 561(1976), 17-28.



Summary Rept.  
AFOSR Grant No. 76-2879

Work in preparation includes a monograph text:

"Introduction to Singular Perturbation Problems for Partial  
Differential Equations"